A game-theoretic approach to cooperation in Multi-Agent Systems

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ABSTRACT
The development of semantic agent frameworks is strongly influenced by the means for representing and reasoning about knowledge. Nevertheless, when faced with decision-making in multi-agent environments, current approaches are limited in their ability to model strategic interactions. As a result, ontological approaches such as those based on OWL, offer only a reactive behavior for such interactions. We argue that a game-theoretic approach is more suitable when faced with modeling cooperation in Multi-Agent Systems. We introduce a new type of games, Boolean Games with Currency, which combine features of Boolean Games, with the transferable payoff setting. We also give a computational characterisation for a solution concept for coalitional stability: the core. We show that the core of a Boolean Game with Currency is always non-empty, and we prove the core membership problem to be co-NP complete.

Categories and Subject Descriptors
H.1 [Models and principles]: General; F.2 [Analysis of algorithms and problem complexity]: General—complexity measures

General Terms
Theory

1. INTRODUCTION
The success of a Multi-Agent System (MAS) is strongly affected by the way decision-making is approached. In traditional semantic systems, agents are treated as inanimate entities that interact with the environment in a strictly reactive way, by responding to external stimuli with particular actions, in order to achieve some goals. In frameworks that support agent cooperation such as [1], the emphasis falls on the knowledge representation means, and on developing a suitable agent platform. Semantic learning methods such as [8] or [7], focus on achieving a certain more-or-less fixed behavior, using machine learning techniques that do not require an in-depth understanding of the underlying processes. All these approaches ignore the fact that, in many situations, the decision to act in a certain way is influenced by the decisions of other agents. To illustrate this, consider the following example: agents 1, 2 and 3 are travelling together, and must decide on the transportation means. The possible options are (a) airplane, (b) bus, and (t) train. The decision is made by a simple voting procedure. If there is a tie, the fastest form of transportation wins. Agents have the following preferences (we use $x \succ_i y$ to denote that agent $i$ prefers $x$ to $y$): $a \succ_1 b \succ_1 t$, $b \succ_2 a \succ_2 t$, $t \succ_3 b \succ_3 a$. If each agent votes for his most preferred candidate, then the winner would be $a$, since there is a tie and it is assumed that airplanes are faster than buses or trains. This outcome is highly undesirable for agent 3, who least prefers travelling by air. Nevertheless, if 3 would observe the preferences of 1 and 2, then, he could change his vote from $t$ to $b$, and therefore change the outcome of the voting procedure from $a$ to $b$. Even if $b$ is not 3’s most desirable travelling means, for him it is better than $a$. In this latter case, 3 behaves in a strategic way: his decision is based both on his goal, as well as on the others’ decisions.

By strategic interaction, we refer to any situation where: the state of an agent is dependent both on the environment as well as on the states of other agents, each agent is aware of this fact and exploits it to his own advantage. In the following, we are interested in Multi-Agent Systems where any interactions are done in accordance with these properties. We also make the following assumptions on the characteristics of agents: (i) they are rational, (ii) self-interested and (iii) have limited resources.

Assumption (i) refers to agents that are fully aware of the actions available for them as well as their consequences and have well-defined preferences over all possible states-of-the world. By (ii) we understand that each agent has a certain preferred state-of-the-world, and would take any possible action to achieve this state, regardless of the interests of other agents. Assumption (iii) states that any agent can
only use some finite computational effort, in order to derive
the proper actions he should undertake.

Game Theory (GT) is a powerful tool in the design of any
kind of strategic interaction between agents in a MAS. It
uses the following elements: a set of actions available for
each agent, a set of consequences that result from performing
some actions, a consequence function that, for each action,
associates a proper consequence, and finally, for each agent,
a preference relation over the set of consequences. GT distin-
guishes between two modeling approaches: non-cooperative,
and cooperative. In the non-cooperative setting the empha-
sis falls on the actions available to the individual, as well
as his preferences. The cooperative setting, still preserves
the preferences of individuals but the emphasis now falls on
what agents can jointly achieve.

In the following we introduce a new type of game, suitable
for representing and reasoning about compromise. To our
knowledge, there are no actual applications of game the-
ory in semantic agent frameworks, although game theory is
briefly mentioned in papers such as [5] or [4]. The rest of
the paper is structured as follows: In Section 2 we introduce
a new type of game, Boolean Games with Currency, which
combines the features of Cooperative Boolean Games, with
the transferable payoff setting. In Section 3 we give some
complexity bounds for the computation of the core and fi-
nally, in Section 4 we make some final remarks and sketch
future directions concerning our approach.

2. GAMES WITH CURRENCY
We propose a new type of cooperative games inspired from
[2], where agents are able to specify their goals as proposi-
tional formulae. The difference between our approach and
Cooperative Boolean Games (CBG) [2] is that we consider
situations in which agents are able to exchange payoff. As
a result, our setting combines features of CBG with that of
cooperative games with transferable payoff [6].

Let \( V = \{p, q, r, \ldots\} \) be a finite set of propositional vari-
ables, and \( \mathcal{L}_V \) be the standard propositional logic consisting
of formulas built using variables from \( V \), the negation op-
erator \( \neg \) and the connectives \( \land \) and \( \lor \). In a Cooperative
Boolean Game, each agent \( i \) controls a subset of variables
\( V_i \subseteq V \). \( V_1, V_2, \ldots, V_n \) is a proper partition of \( V \) such
that no two agents can share control over a variable. The possible
actions available to agents consist of setting truth values to
the variables they control. Each action has a certain cost.

The agent \( i \)'s objective is expressed as a boolean formula
\( \gamma_i \in \mathcal{L}_V \).

**Definition 1.** A boolean game with currency (BGC) is
a tuple \( \mathcal{B} = (\mathcal{N}, V, (V_i)_{i \in \mathcal{N}}, (\gamma_i)_{i \in \mathcal{N}}, \mu, c) \) where:

- \( \mathcal{N} = \{1, 2, \ldots, n\} \) is the set of agents;
- \( V \) is a finite set of propositional variables;
- \( V_1, V_2, \ldots, V_n \) is a partition of \( V \). Each \( V_i \) denotes
  the set of variables controlled by agent \( i \);
- \( \mu : \mathcal{N} \to \mathbb{R}_{\geq 0} \): \( \mu \) assigns a value to an agent's goal
  formula; in the following, we use \( \mu \) as a shorthand for
  the value \( \mu(i) \).
- \( \gamma_1, \gamma_2, \ldots, \gamma_n \in \mathcal{L}_V \) are the goals of each agent;
- \( c : \mathcal{N} \times V \times \mathcal{B} \to \mathbb{R}_{\geq 0} \) is a function assigning for each
  agent, variable and truth value, a cost of setting that
  particular truth value. We use \( c_i(w, t) \) as a shorthand
  for \( c(i, w, t) \).

**Example 1 (BGC).** Consider the following scenario:
Jim, Tom and Harry are colleagues and, occasionally, they
go out together. They can choose between a restaurant and
a pub. Jim would like to eat at the restaurant, but he is
looking forward to spend more time with Harry. Harry is
only interested to go to the pub and have some beers, and he
is not interested in company. Like Jim, Tom is also indif-
ferent about the location, and doesn't want to go out alone.
But Jim finds Tom to be a really boring person, and prefers
the company of Harry. This scenario is modeled by a BGC
having:

- \( \mathcal{N} = \{1, 2, 3\} \) - the agents, Jim, Harry and Tom
- \( V = \{p_1, p_2, p_3, r_1, r_2, r_3\} \) and the following partition:
  for all \( i \in \{1, 2, 3\} \) \( V_i = \{p_i, r_i\} \) model the agent \( i \)'s
  possible choices: \( p_i \) and \( r_i \) models agent \( i \)'s choice: go-
ing to the pub, or restaurant, respectively.
- \( c = 0 \) is a constant cost function;
- \( \mu_1 = 6, \mu_2 = \mu_3 = 2 \);
- \( \gamma_1 = (p_1 \land p_2 \land \neg p_3) \lor (r_1 \land r_2 \land \neg r_3) \);
- \( \gamma_2 = p_2 \land \neg r_2 \);
- \( \gamma_3 = (p_3 \land p_2) \lor (r_3 \land r_2) \);

We now look at the possible outcomes of BGC and, given
a certain outcome, we define a measure of fulfillment for
each agent, i.e. a utility. For an arbitrary set of variables
\( V \subseteq 2^\mathcal{V} \), let \( \xi = (\xi_w)_{w \subseteq \mathcal{V}} \) be a \( V \)-valuation giving truth
assignments for variables in \( V \). If \( w \) is a variable in \( V \), then
either \( \xi_w = \top \) (true) or \( \xi_w = \bot \) (false). For all agents \( i \), we
define a function testing the entailment of \( i \)'s goal:
\[
1_i(\xi) = \begin{cases} 
1 & \text{if } \xi \models \gamma_i \\
0 & \text{otherwise}
\end{cases}
\]

The utility of an agent can be defined in the spirit of [2]. If
\( \xi \) is a \( V \)-valuation, then:
\[
u_i(\xi) = 1_i(\xi) \ast \mu_i - \sum_{w \in V \land w_i} c_i(w, \xi_w)\]

Notice that, as opposed to CBG [2], in our setting it might
be the case that the utility of an agent is not positive when
his goal is satisfied. Such a condition could be achieved only
by adding restrictions to the value and cost functions \( \mu \) and
c. Therefore, it is not always the case that an agent prefers
an outcome where his goal is satisfied.

**Proposition 1.** Cooperative Boolean Games are a speci-
cal case of Boolean Games with Currency.
Proof. Consider a game $\mathfrak{B} = (N, V, (V_i)_{i \in N}, (\gamma_i)_{i \in N}, \mu, c)$, with the following restrictions on $\mu$ and $c$:

- $\forall w \in V, c_i(w, \bot) = 0$; only setting variables to true inflicts a cost on agents;
- $\forall w \in V, c_i(w, \top) = c_i(w, \bot)$; the costs for setting a variable to true are the same for all agents;
- $\mu_i > \sum_{w \in V} c_i(w, \top)$; the value of each agent’s goal is strictly larger than the sum of all the costs for variables he controls.

The last restriction ensures positive values for utilities $u_i$, if agents satisfy their goals. Then, $u_i$ can induce a preference relationship over valuations $\xi$ such that: (i) between a valuation that does satisfy his goal and another that does not, a agent will always choose the former, and (ii) between two valuations that either both satisfy or do not satisfy his goal, a agent will chose the one minimising costs. □

3. THE CORE OF A BGC

In Example 1, it is obvious that goal satisfaction is dependent on a certain deal of cooperation between agents. In the following we formally define how cooperation between agents occurs.

For a game $\mathfrak{B} = (N, V, (V_i)_{i \in N}, (\gamma_i)_{i \in N}, \mu, c)$, and a coalition $S \subseteq N$, the endowment of each agent $i \in S$ is:

$$1_i(\xi)\mu_i - \sum_{w \in V \setminus V_i} c_i(w, \xi_w) \quad (1)$$

The endowment is computed by taking the value of the satisfied goal, if it is indeed satisfied, and subtracting the involved costs. Based on the endowment, the characteristic function $v$ is defined as follows:

$$v(S) = \max_{\xi = (\xi_i)_{i \in V}} \sum_{i \in S} 1_i(\xi)\mu_i - \sum_{w \in V \setminus V_i} c_i(w, \xi_w) \quad (2)$$

Equation 2 describes the worth of a coalition $S$ as being the maximum sum of all endowments, obtained by some valuation $\xi$.

The cf form $(N, v)$ of a BGC $\mathfrak{B}$ is obtained by taking $N$ to be the number of agents of $\mathfrak{B}$, and by computing the characteristic function $v$ according to Equation 2. The core of a BGC expressed in cf form $(N, v)$ is the set of all feasible payoff profiles $(x_i)_{i \in N}$, for which there is no coalition $S$ such that $v(S) > x(S)$. We say that $(x_i)_{i \in N}$ is group rational: no other coalition $S$ has an incentive to deviate.

Example 2 (Core). Going back to Example 1, it is straightforward that there is no valuation able to satisfy all goals. A stable valuation is $\xi = (p_1, p_3)$, which leaves agent Jim unsatisfied, but here he cannot object, since there is no coalition and $\xi$ that can guarantee $\neg p_3$ for him. But, since Jim’s goal satisfaction is measured by a particular real value $\mu_1$, and since this value can be transferred between agents, then he could change the outcome of the game, by tipping off Tom and thus determining him not to come to the bar. This would be possible under the following assumption: Jim’s happiness when going with Harry is much larger than Tom’s, and therefore he can afford to tip off Tom, and make him earn more by staying at home (or going to the restaurant), than by coming to the pub to see Harry.

This gives us the intuition that, in settings such as ours, agents might attempt to maximise their utilities by means other than satisfying goals. One such mean is by participating in coalitions where some agents have goals with great values. Agents contribute to these goals instead of their own, and, in certain settings, can achieve more value. This is the case for Tom, who could participate in forming the grand coalition by fulfilling Jim’s goal instead of his own, under the condition that he can obtain a higher value this way.

Proposition 2 (Core membership). The core membership problem for BGC is co-NP complete.

Proof (sketch). Core membership is a decision problem $\text{MEM}(\mathfrak{B}, x)$ which, given a Boolean Game with Currency $\mathfrak{B}$ and a vector $x$ of size $N$, asks whether $x$ is in the core of $\mathfrak{B}$. Recall the definition of core membership, which requests that there is no coalition $S \subseteq N$ and payoff profile $y$ such that $S$ prefers $y$ over $x$. Now, consider the complement of core membership problem $\text{MEM}(\mathfrak{B}, x)$ which asks if there exists a coalition $S$, and a payoff $y$ such that $y(S) > x(S)$. An equivalent definition for this problem, is asking whether there exists a coalition $S \subseteq N$ such that $v(S) > x(S)$. If such an $S$ exists, then there is also an $S$-feasible payoff profile $y$, which agents in $S$ will prefer over $x$.

In the following, we prove $\text{MEM}(\mathfrak{B}, x)$ to be NP-complete.

The membership $\text{MEM}(\mathfrak{B}, x)$ is in NP is straightforward: A procedure can build all coalitions $S$ in nondeterministic polynomial time, and, for each $S$, checking whether $v(S) > x(S)$ holds can be done in deterministic polynomial time. The NP-hardness of $\text{MEM}(\mathfrak{B}, x)$ is due a reduction from the k-VERTEX-COVER(G) problem.

Let $G = (A, E)$ be a graph. k-VERTEX-COVER(G) asks if there is a subset $B \subseteq A$ of vertices, with $|B| = k$, such that all edges from $E$ are covered by at least one vertex. Starting from an instance of k-VERTEX-COVER(G), we build an instance of $\text{MEM}(\mathfrak{B}, x)$ in the following way:

- for each vertex $a \in A$, we create a agent $n_a \in N$. We add an additional agent $n_0$ to $N$;
- for each edge $e = (a, b) \in E$, we create two variables $p_a, p_b \in V$, and subsequently assign their control to the corresponding agents: $p_a \in V_{n_a}$ and $p_b \in V_{n_b}$ (each agent controls one “side” of an edge). For the additional agent, we have $V_{n_0} = \emptyset$;
- we define the cost function $c$ such that $\forall i, \forall p \in V, c_i(p, \bot) = 1$ and $c_i(p, \top) = 0$;
- the goal of each agent $n_a$ is $\gamma_{n_a} = \bigwedge_{e = (a, b)} (p_a \lor p_b)$ ($n_a$’s goal is satisfied when at least one “side” of each of his
incident edges is set to \( \top \)). The goal for agent \( n_0 \) is \( \gamma_{n_0} = \bigwedge_{e=(a,b) \in E} (p_a \lor p_b) \) (it is satisfied when all edges have at least one side set to \( \top \));

- the goal value for each agent \( n_a \) is \( \mu_{n_a} = 0 \). The goal for agent \( n_0 \) is \( \mu_{n_0} = 2|E| \);
- we build a payoff vector \( x \) such that \( x_{n_a} = |E| - 1/2 \) and \( \forall n_a, x_{n_a} = \frac{1}{2(k+1/2)} \).

We only show the first part of the double implication. The second part follows a similar argument. Suppose \( W \) is a vertex cover of size \( k \). Then, the coalition \( S_W = \{ n_0 \} \cup \{ n_a | a \in W \} \) will achieve the maximum value under any valuation \( \xi = (\xi_i)_{i \in S_W} \) such that, \( \forall e = (a, b) \xi_{p_e} = 1 \) and \( \xi_{p_0} = 0 \) (exactly one “side” of each edge is set to true). Under any such valuations, all agents see their goals satisfied. Therefore:

\[
v(S_W) = \sum_{i \in S_W} \mu_i - \sum_{i \in S_W} \sum_{w \in V \setminus w_i} c_i(w, \top) = 2|E| - |E| = |E|
\]

Since, according to the above construction, the payoff profile \( x \) associated to \( S_W \) yields:

\[
x(S_W) = x_{n_0} + k + \frac{1}{2(k+1/2)} = \frac{|E| - 1/2 + \frac{k}{2} + 1}{2} \geq \frac{|E| - 1}{2k + 1} < |E| = v(S_W)
\]

then \( x \) together with \( S_W \) is a credible deviation to the grand coalition.

**Proposition 3.** Given a BGC in cf-form, the characteristic function \( v \) is super-additive, that is for every \( S_1, S_2 \subseteq N \) such that \( S_1 \cap S_2 = \emptyset \), the following holds: \( v(S_1 \cup S_2) \geq v(S_1) + v(S_2) \).

**Proof.** Let \( S_1, S_2 \) be two disjoint coalitions and \( \xi_1, \xi_2 \) be the solutions of Equation 2 giving the values \( v(S_1) \) and \( v(S_2) \), respectively. Let \( V_1 = \bigcup_{i \in S_1} V_i \) and \( V_2 = \bigcup_{i \in S_2} V_i \). It is straightforward from the definition of a BGC that variable sets \( V_1 \) and \( V_2 \) are disjoint (since no two agents can have control over the same variable). Since \( \xi_1 \) and \( \xi_2 \) contain truth assignments for every variable in \( V_1 \) and \( V_2 \) respectively, then \( \xi_1 \) and \( \xi_2 \) cannot assign different values to the same variable. Then, for all goals \( \gamma_1 \) and \( \gamma_2 \) satisfied by \( \xi_1 \) and \( \xi_2 \) respectively, it is the case that \( \xi_1 \cup \xi_2 = \gamma_1 \) and \( \xi_1 \cup \xi_2 = \gamma_2 \). Therefore, each satisfied goal in \( S_1 \) or \( S_2 \) is satisfied also in \( S_1 \cup S_2 \) (possibly cheaper). As a result we have that \( v(S_1 \cup S_2) \geq v(S_1) + v(S_2) \). \( \square \)

**Proposition 4.** The core of a BGC is non-empty.

**Proof.** This is a direct consequence of the property of super-additivity of BGC. First of all, notice that the definition of the core from Section \( 2 \) can be reformulated as:

\[
\text{core}(\mathfrak{B}) = \{ x \mid \forall S \subseteq N : x(S) \geq v(S) \}
\]

Now, since \( \mathfrak{B} \) is super-additive, it follows that \( \forall S \subseteq N \), \( v(N) \geq v(S) \), since \( N = S \cup N \setminus S \) and \( v(N) \geq v(S) + v(N \setminus S) \). Then, any feasible payoff profile that offers a division of \( v(N) \) is in the core of \( \mathfrak{B} \). \( \square \)

### 4. Conclusions and Future Work

The game-theoretic setting proves to be a suitable direction for studying agent interaction and cooperation. The results we describe, the computational complexity for the core membership and core emptiness problems are inherently theoretic, but not without applicability. Although no quite optimistic (the core membership is shown to be co-NP-complete), the results suggest that, for small games, computing the core is possible, and could be implemented in systems that assist humans in decision-making. Also, iterative techniques, in the spirit of [1], for deriving a more efficient computation process should be researched. Following the line of [3], the goal value function from BGC’s can be used to incentivise certain behavior within a group of agents. We are currently tackling these ideas, as well as attempting to produce an efficient implementation, based on our results.

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### 6. References


